Transmission Modes in a Braided Coaxial Cable and Coupling to a Tunnel Environment

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***Abstract*—Radio frequency transmission in a semicircular tunnel containing a braided coaxial cable is considered. The general formulation accounts for both the ohmic losses in the tunnel wall and a thin lossy film layer on the outer surface of the dielectric jacket of the cable. Using a quasi-static approximation, it is found that the propagation constants of the low-frequency transmission line modes are obtained through the solution of a cubic equation. However, for the special case when the conductivity thickness product of the lossy film layer vanishes, this cubic equation reduces to a quadratic. The spatially dispersive form of the braid transfer impedance is also accounted for. It is shown that the quasistatic theory is well justified for frequencies as high as 100 MHz for typical tunnel geometries. Finally, special characteristic impedances are derived for the various modes of the equivalent multiconductor transmission line.**

I. Introduction

A COAXIAL CABLE that allows continuous leakage through its outer sheath can be exploited for continuous-access guided communications [1]. The environment, of course, plays an important role whether it be an adjacent roadway, railway right-of-way, or tunnel. A key aspect of such systems is the coupling (intentional or nonintentional) between the transmission modes within the cable and the transmission mode(s) in the external region. Before an optimum communication system can be designed to utilize these coupled modes, a better understanding of the propagation mechanisms is needed.

In this paper, we consider specifically a semicircular tunnel model with a coaxial cable whose sheath is characterized by a transfer impedance. We also allow for the presence of a lossy film on the outer surface of the concentric dielectric jacket. For applications in mine environments such a lossy layer can represent the effect of a thin conducting fluid (i.e., saline water) or conductive dust. Of course, the ohmic losses in the curved tunnel walls are also included.

The main objective is to obtain specific results for the attenuation rates of the dominant transmission modes and to provide convenient definitions for the corresponding characteristic impedances. We also demonstrate the validity of the quasi-static formulation of the problem by making a comparison with calculations based on a more general (and more complicated) mode equation.

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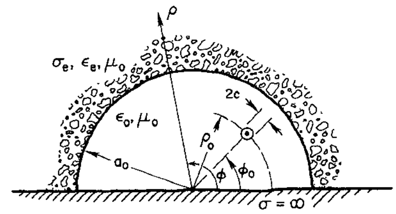


Fig. 1. Cross-section of semicircular tunnel containing coaxial cable (not to scale).

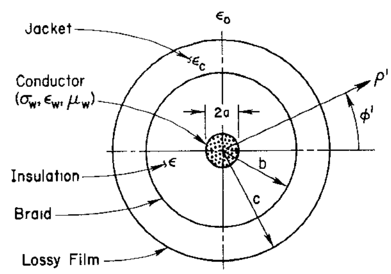


Fig. 2. The braided coaxial cable.

II. Formulation

The model assumed is described in terms of a cylindrical coordinate system (ρ, ϕ, z) and is shown in Fig. 1. The tunnel wall is located at $\rho = a_0$ for $0 < \phi < \pi$, and the assumed perfectly conducting tunnel floor is located at $\phi = 0$ and $\phi = \pi$ for $0 < \rho < \infty$. The region defined by $\rho > a_0$ and $0 < \phi < \pi$ is a homogeneous medium with conductivity σ_e and permittivity ϵ_e. The coaxial cable with outside radius c is centered at $\rho = \rho_0$ and $\phi = \phi_0$. The region defined by $\rho < a_0$, $0 < \phi < \pi$, and $\rho' > c$, where ρ' is the radial component of a cylindrical coordinate system (ρ', ϕ', z) centered at (ρ_0, ϕ_0), is described by the free space permittivity and permeability ϵ_0 and μ_0, respectively. The geometry of the coaxial cable is shown in Fig. 2. The inner conductor has radius a and high but finite conductivity σ_w. The surrounding insulation of radius b is a lossless dielectric with permittivity ϵ. The braided sheath located at $\rho' = b$ is characterized by a surface transfer impedance Z_T. The outer dielectric coating has radius c and permittivity ϵ_c. We also allow for the possibility that a thin lossy film is located at $\rho' = c$ which is characterized by a transfer impedance Z_L. We assume that the fields of each mode of this structure vary as $\exp(-\Gamma z + i\omega t)$ where ω is the angular frequency, and Γ is the complex propagation constant for the particular mode.

In a previous paper, Hill and Wait [2] obtained a modal solution for the similar problem of a single conductor trolley wire in a semicircular tunnel. This solution is general for any thin wire or cable that can be characterized by a series impedance. Their mode equation is given here in a more convenient form for our purpose:

$$K_0(vc)-K_0(v\rho_d)-S+\frac{2\pi\gamma_0}{v^2\eta_0}(Z(\Gamma)+Z_e)=0 \quad (1)$$

where

$$\gamma_0^2=-\epsilon_0\mu_0\omega^2=-(2\pi/\lambda_0)^2=-k_0^2, \quad \rho_d=2\rho_0\sin\phi_0$$

$$v^2=\gamma_0^2-\Gamma^2, \quad \eta_0=(\mu_0/\epsilon_0)^{1/2}=120\pi$$

where $Z(\Gamma)$ is an effective impedance of the cable that we define below. The factors S and Z_e account for the presence of the tunnel walls and are given by

$$S=2\sum_{m=1}^{\infty}\frac{K_m(va_0)}{I_m(va_0)}I_m(v\rho_0)I_m(v(\rho_0+c))[1-\cos 2m\phi_0] \quad (2)$$

and

$$Z_e=-\frac{v^2\eta_0}{\pi\gamma_0}\sum_{m=1}^{\infty}(R_m-1)\frac{K_m(va_0)}{I_m(va_0)} \cdot I_m(v\rho_0)I_m(v(\rho_0+c))[1-\cos 2m\phi_0] \quad (3)$$

where

$$R_m=\frac{[(\gamma_0/v)K_m'(va_0)/K_m(va_0)]+Y_m\eta_0+\delta_m\eta_0}{[(\gamma_0/v)I_m'(va_0)/I_m(va_0)]+Y_m\eta_0+\delta_m\eta_0}$$

$$Y_m=\left(\frac{i\gamma_e^2}{u\mu_0\omega}\right)\frac{K_m'(ua_0)}{K_m(ua_0)}$$

$$\delta_m\eta_0=\frac{(im\Gamma/a_0)^2(v^{-2}-u^{-2})^2}{[(\gamma_0/v)I_m'(va_0)/I_m(va_0)]+Z_m/\eta_0}$$

$$Z_m=-\left(\frac{i\omega\mu_0}{u}\right)\frac{K_m'(ua_0)}{K_m(ua_0)}$$

$$\gamma_e^2=i\omega\mu_0(\sigma_e+i\omega\epsilon_e) \quad u=(\gamma_e^2-\Gamma^2)^{1/2}.$$

Note that S represents the reflected field from perfectly conducting cylindrical walls and thus Z_e accounts for the ohmic losses due to the finite conductivity of the tunnel walls.

Previously, Wait and Hill [3] derived an effective series impedance $Z(\Gamma)$ for a braided coaxial cable. In general, the resulting expression for $Z(\Gamma)$ is quite complicated. However, they show that, if some quasi-static approximations are invoked, $Z(\Gamma)$ can be expressed approximately by the simpler form where

$$Z(\Gamma)=\frac{Z_L(Z_c+Z_b)}{Z_L+Z_c+Z_b}$$

$$Z_b=\frac{Z_T(Z'+Z_i)}{Z_T+Z'+Z_i} \quad (4)$$

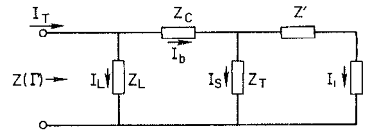


Fig. 3. The equivalent network which yields the effective series impedance of the cable in the quasi-static approximation.

$$Z_i=\frac{\eta_0\gamma_0}{2\pi a\gamma_w}\frac{I_0(\gamma_w a)}{I_1(\gamma_w a)} \qquad \gamma_w^2=i\sigma_w\mu_0\omega$$

$$Z'=-(k^2+\Gamma^2)\alpha' \qquad k^2=\omega^2\mu_0\epsilon$$

$$Z_c=-(k_c^2+\Gamma^2)\alpha_c \qquad k_c^2=\omega^2\mu_0\epsilon_c$$

$$Z_L=[2\pi c(\sigma d)]^{-1}$$

$$\alpha'=(2\pi i\omega\epsilon)^{-1}\ln(b/a) \text{ and } \alpha_c=(2\pi i\omega\epsilon_c)^{-1}\ln(c/b).$$

Here (σd) is the conductivity–thickness product of the thin lossy layer that may be present.

The equivalent circuit for $Z(\Gamma)$ is the network shown in Fig. 3. The general spatially dispersive form of the sheath transfer impedance Z_T can be employed here. An approximate Γ-dependent form [4],[5] appropriate for the present analysis is

$$Z_T=Z_0+\alpha_t\Gamma^2$$

where

$$\alpha_t=2Z_0(k^2+k_c^2)^{-1}\cos^2\psi$$

and $Z_0=i\omega L_T$ where L_T is the static sheath transfer inductance, and ψ is the braid weave angle. Note that α_t accounts for the spatial dispersion of the sheath transfer impedance.

The proper mode equation for our problem is obtained by using (4) in (1). Normally, the numerical solution for the complex propagation constants Γ for the various modes of the structure are obtained by employing an iterative procedure for locating complex roots [3], [5]. However, here we take a different approach and we use an iterative solution of (1) only for purposes of comparison.

III. Quasi-Static Mode Equation

Solution of (1) for the modal propagation constants can be simplified if some quasi-static approximations are invoked. Specifically, we assume that $|\gamma_e/\gamma_0|^2\gg 1$ and that $|va_0|\ll 1$. In fact, using a previous result of Wait [6] and small argument approximations for the modified Bessel functions, we can represent (2) and (3) by

$$S\approx\sum_{m=1}^{\infty}\frac{r^m}{m}(1-\cos 2m\phi_0) \quad (5)$$

and

$$Z_e\approx\frac{2\gamma_0\eta_0}{\pi\gamma_e a_0}\sum_{m=1}^{\infty}r^m\frac{K_m(\gamma_e a_0)}{K_{m+1}(\gamma_e a_0)}\sin^2 m\phi_0 \quad (6)$$

where

$$r=\rho_0(\rho_0+c)/a_0^2.$$

Note that both S and Z_e are independent of Γ with this approximation. If we now use

$$\sum_{n=1}^{\infty} \frac{a^n}{n} \cos nx = -\frac{1}{2} \ln\left[1 - 2a\cos x + a^2\right], \qquad \text{for } a^2 < 1$$

given by Wheelon [7] in (5), we find that (1) becomes

$$v^2 \ln R + N(Z(\Gamma) + Z_e) = 0 \tag{7}$$

where

$$R = 2\rho_0 \sin\phi_0 c^{-1}(1-r)(1 - 2r\cos\phi_0 + r^2)^{-1/2}$$

and

$$N = 2\pi\gamma_0/\eta_0.$$

After considerable algebraic manipulation, (7) becomes a cubic equation of the form

$$a_3(\Gamma^2)^3 + a_2(\Gamma^2)^2 + a_1(\Gamma^2) + a_0 = 0 \tag{8}$$

where

$$a_3 = q_2(\ln R)/Z_L \tag{9}$$

$$a_2 = Nq_2 - s_1 \ln R + \left[(\gamma_0^2 q_2 - q_1)\ln R + NZ_e q_2\right]/Z_L \tag{10}$$

$$a_1 = N(Z_e s_1 + q_1) + (\gamma_0^2 s_1 - s_0)\ln R + \left[(\gamma_0^2 q_1 - q_0)\ln R + NZ_e q_1\right]/Z_L \tag{11}$$

$$a_0 = N(Z_e s_0 + q_0) + \gamma_0^2 s_0 \ln R + q_0(\gamma_0^2 \ln R + NZ_e)/Z_L \tag{12}$$

and

$$q_0 = (k^2\alpha' - Z_i)(k_c^2\alpha_c - Z_0) - k_c^2\alpha_c Z_0$$

$$q_1 = (\alpha_c - \alpha_t)(k^2\alpha' - Z_i) + \alpha'(k_c^2\alpha_c - Z_0) - \alpha_c(Z_0 + k_c^2\alpha_t)$$

$$q_2 = \alpha'(\alpha_c - \alpha_t) - \alpha_t\alpha_c$$

and

$$s_0 = Z_i + Z_0 - k^2\alpha' \qquad s_1 = \alpha_t - \alpha'.$$

Now (8) is a cubic equation and it has three roots which can be solved for algebraically. Two of these roots can be identified as the propagation constants of the well-known monofilar and bifilar modes. But, in addition, there is a third mode which we will refer to as the jacket mode since it is found that in this mode, the current and return current flows primarily in the braided sheath and the lossy film layer, respectively, and that the other currents are negligible. Examination of (9) reveals that in the absence of the outer lossy film ($Z_L \to \infty$), a_3 vanishes, and (8) reduces to a quadratic equation in Γ^2. Consequently, there are now only two solutions. As one would expect, the jacket mode has disappeared, leaving only the monofilar and bifilar modes.

These results are not unexpected. In the general case, we have four axial conductors (the tunnel wall and floor, the cable's inner conductor, the braided sheath, and the lossy film layer). Thus, at sufficiently low frequencies, there will be three TEM modes [8]. But, of course, in the absence of the lossy film, there remain only three conductors and thus only two TEM modes would exist in this low frequency limit. When ohmic losses in these conductors are considered, we still expect three (or two, if $\sigma d = 0$) dominant modes provided the tunnel cross section is small compared with the free space wavelength λ_0.

IV. Characteristic Impedances for Multiconductor Transmission Line

Because the dominant low-frequency modes of this system behave like TEM transmission-line modes, it is useful to treat the problem as a multiconductor transmission line using transmission-line concepts. Such an approach proves useful, for example, in considering the mode conversion problem resulting from the presence of axial nonuniformities in the cable or tunnel. Thus we need to derive the characteristic impedance of each line for each quasi-TEM mode. Specifically, we define

$$K_L = \frac{V_L}{I_L e^{-\Gamma z}}, \qquad K_s = \frac{V_s}{I_s e^{-\Gamma z}}, \qquad K_i = \frac{V_i}{I_i e^{-\Gamma z}} \tag{13}$$

as the impedance associated with the lossy film, the braided sheath, and inner conductor, respectively. The voltages V_L, V_s, and V_i are defined by

$$V_L = -\int_0^{\rho_0 - c} E_\rho(\phi = \phi_0)\, d\rho$$

$$V_s = V_L + \int_b^c E_{\rho'}\, d\rho'$$

$$V_i = V_s + \int_a^b E_{\rho'}\, d\rho' \tag{14}$$

and are the voltages of the lossy film, the braid, and the inner conductor, respectively, with reference to the tunnel floor. Similarly, I_L, I_s, and I_i are the currents in each conductor (see Fig. 3).

It is not difficult to show that within the quasi-static approximation, E_ρ can be determined from

$$E_\rho = \partial^2 U/\partial\rho\partial z = -\Gamma \partial U/\partial\rho \tag{15}$$

where U has the form

$$U \approx \frac{\eta_0}{2\pi\gamma_0} e^{-\Gamma z} I_t \left\{ \ln\frac{\rho_0^-}{\rho_0^+} - \sum_{m=1}^{\infty} \frac{1}{m}\left(\frac{\rho\rho_0}{a_0^2}\right)^m \cdot \left[\cos m(\phi - \phi_0) - \cos m(\phi + \phi_0)\right] \right\} \tag{16}$$

and

$$\rho_0^+ = \left[\rho_0^2 + \rho^2 - 2\rho\rho_0 \cos(\phi - \phi_0)\right]^{1/2}$$

$$\rho_0^- = \left[\rho_0^2 + \rho^2 - 2\rho\rho_0 \cos(\phi + \phi_0)\right]^{1/2}$$

$$I_t = I_L + I_s + I_i.$$

Similarly, we find that the radial electric field within the cable using these quasi-static approximations is given by

$$E_{\rho'} \approx \begin{cases} \dfrac{\Gamma I_i e^{-\Gamma z}}{2\pi i\omega\epsilon\rho'}, & a < \rho' < b \\[2ex] \dfrac{\Gamma I_b e^{-\Gamma z}}{2\pi i\omega\epsilon_c\rho'}, & b < \rho' < c \end{cases} \tag{17}$$

where $I_b = I_s + I_i$. Combining (13)–(17) and using the summation form used earlier, we obtain

$$K_L=\left(\frac{1+p+q}{q}\right)\frac{\Gamma \ln R}{N}$$

$$K_s=\Gamma\left[\left(\frac{1+p+q}{p}\right)\frac{\ln R}{N}+\left(\frac{1+p}{p}\right)\alpha_c\right] \tag{18}$$

and

$$K_t=\Gamma\left[(1+p+q)\frac{\ln R}{N}+(1+p)\alpha_c+\alpha'\right]$$

where the current ratios p and q are defined by

$$p\equiv\frac{I_s}{I_t}=\frac{Z'(\Gamma)+Z_t}{Z_T(\Gamma)} \tag{19}$$

and

$$q\equiv\frac{I_L}{I_t}=(1+p)\frac{Z_b(\Gamma)+Z_c(\Gamma)}{Z_L}.$$

The characteristic impedances defined by (18) are dependent upon Γ and consequently they assume different values for each mode. Thus we have $K_L^{(j)}$, $K_s^{(j)}$, and $K_t^{(j)}$ corresponding to the propagation constant of each mode (Γ_j) for $j=1,2,3$. In the special case when there is no lossy film ($\sigma d=0$), I_L vanishes, and we consider only the two characteristic impedances $K_s^{(j)}$ and $K_t^{(j)}$ for $j=1,2$.

V. Some Numerical Results

We now present some examples of numerical results for typical configurations of the geometry shown in Figs. 1 and 2. In all examples that follow, we consider a tunnel with radius $a_0=2m$ and rock conductivity and permittivity of $\sigma_e=10^{-2}$ mhos/m and $\epsilon_e=10\epsilon_0$, respectively. The dimensions of the cable are as follows: $a=1.5$ mm, $b=10$ mm, and $c=11.5$ mm. The permittivities of the cable insulation and jacket are given by $\epsilon=2.5\epsilon_0$ and $\epsilon_c=3.0\epsilon_0$, which are typical values. Finally, in all cases, the cable is located at $\phi_0=45°$, and the cable's braid is woven at the angle $\psi=45°$.

In order to ascertain the accuracy of the quasi-static mode equation (7), we first consider a comparison of results computed using this approximation to results computed from (1) directly using iterative techniques. Fig. 4 compares the attenuation rates of the monofilar mode over the frequency range 0.2 to 100 MHz for various values of the cable's radial position ρ_0. (Attenuation rate in dB/km $=8686$. Re (Γ).) Here we take the braid transfer inductance L_T to be 40 nH/m which corresponds to the FONT cable [9], $\sigma_w=5.7\times10^7$ mhos/m, and $\sigma d=0$. A similar comparison for the bifilar mode shows the two solutions to be virtually indistinguishable over this range of frequencies. In addition, if a nonzero σd is introduced, a comparison of attenuation rate of the jacket mode also shows the two solutions to be virtually the same. These comparisons clearly demonstrate the utility of the quasi-static mode equation for typical tunnel parameters at radio frequencies. The remaining examples given here have been computed solely using the quasi-static approximation that is much more economic.

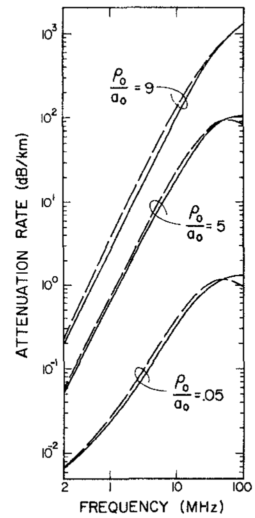
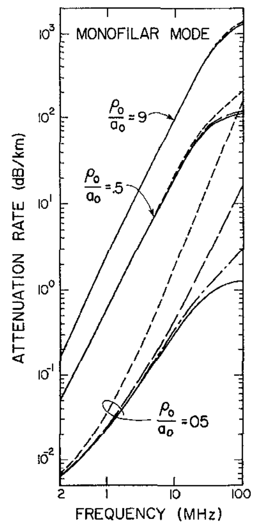


Fig. 4. Attenuation rate of the monofilar mode versus frequency for various values of (ρ_0/a_0). With quasi-static approximation:———. Without quasi-static approximation:------.

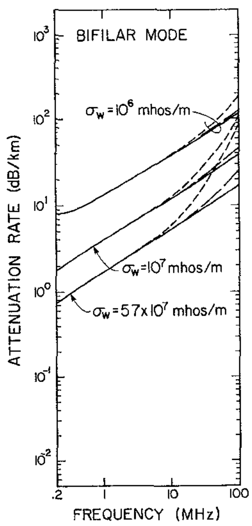
In our next example, we again consider the case when $L_T=40$ nH/m but allow ρ_0, σ_w, and σd to assume various values. These results are shown in Fig. 5(a) and (b). It is found that the monofilar mode is essentially independent of the inner conductor's conductivity (σ_w) whereas the bifilar mode is essentially independent of the cable position within the tunnel. This is in agreement with similar results obtained for a coaxial cable in a circular tunnel [3]. It is interesting to note, however, that the attenuation rates for the monofilar mode (Fig. 5(a)) are significantly lower than those given [3] for similar parameters in a circular tunnel. This is not unexpected since, in the semicircular tunnel, a portion of the return current for the monofilar mode flows in the perfectly conducting tunnel floor and thus suffers less attenuation. On the other hand, there is virtually no difference between the bifilar modes, which are relatively insensitive to the environment outside the cable.

Now we turn our attention to the jacket mode. Fig. 6 shows the attenuation rate of this mode as a function of frequency for various values of L_T and σd. The most immediate observation to be made is that the attenuation rate for this mode is orders of magnitude higher than that of the other two modes. It is also observed that the attenuation is approximately proportional to $(\omega/\sigma d)^{1/2}$.

Another quantity of interest is the normalized phase defined by Im $(\Gamma)/k_0$ which is inversely proportional to the phase velocity with which the mode propagates along the tunnel. Some values of the normalized phase are given in Table I for the monofilar and bifilar modes. Here $L_T=2$ nH/m, $\sigma_\omega=5.7\times10^7$ mhos/m, and $\sigma d=0$. We note that for the monofilar mode, the normalized phase is slightly larger than unity when the cable–tunnel wall separation is large, but increases rapidly as the cable approaches the tunnel wall. On the other hand, the normalized phase of the bifilar mode is essentially independent of this separation and is slightly greater than $k/k_0=(\epsilon/\epsilon_0)^{1/2}=1.581$. This suggests the interesting possibility of adjusting the dielectric constant of the cable insulation



(a)



(b)

Fig. 5. Attenuation rate versus frequency for (a) monofilar mode, (b) bifilar mode. $\sigma d = 0$:———. $\sigma d = 10^{-3}$mhos:— - —. $\sigma d = 10^{-2}$ mhos:———. $\sigma d = 10^{-1}$ mhos:------.

and the cable–tunnel wall separation such that the two modes will propagate with the same phase velocity.

Finally, we consider the normalized phase for the jacket mode as given in Table II for three values of σd. Here $L_T = 2$ nH/m, and the values given are essentially independent of the other parameters. We see that, for typical values of the conductivity–thickness product of the lossy film, the normalized phase is much larger than one, and consequently, the mode propagates very slowly. It should be pointed out that, like this mode's attenuation rate, the phase is essentially proportional to $(\omega/\sigma d)^{1/2}$.

VI. Concluding Remarks

We have obtained a remarkably simple description of the dominant transmission modes for a leaky coaxial cable located in a uniform straight tunnel with a semicircular cross section. The formulation is facilitated by the assumption that the floor of the tunnel, including the effect of any metallic rails, is represented adequately by a perfectly conducting ground plane. This allows us to define characteristic impedances of the transmission modes in a meaningful fashion. In a subsequent study, we shall consider the application of these results to the case where the cable and/or the tunnel environment undergoes a lateral variation such as an abrupt modification of the effective series or shunt impedance.

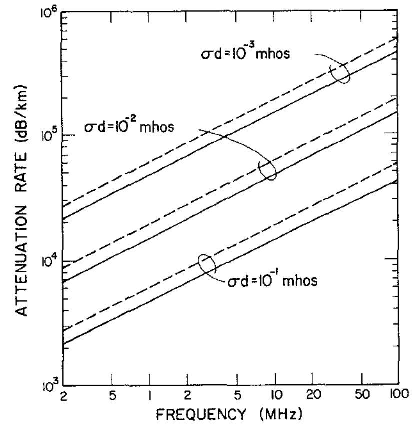


Fig. 6. Attenuation rate of jacket mode versus frequency for various values of σd. $L_T = 40$ nH/m:———. $L_T = 2$nH/m:------.

TABLE I
NORMALIZED PHASE $\mathrm{Im}(\Gamma)/k_0$

f	Monofilar $\rho_o/a_o = .5$	Monofilar $\rho_o/a_o = .9$	Monofilar $\rho_o/a_o = .98$	Bifilar $\rho_o/a_o = .5$
.2	1.058	1.251	1.685	1.604
.5	1.057	1.248	1.680	1.596
1.0	1.056	1.244	1.674	1.593
2.0	1.055	1.238	1.665	1.590
5.0	1.051	1.225	1.644	1.587
10.0	1.047	1.208	1.617	1.586
20.0	1.040	1.176	1.566	1.585
50.0	1.030	1.103	1.444	1.585
100.0	1.027	1.054	1.332	1.584

TABLE II
NORMALIZED PHASE $\mathrm{Im}(\Gamma)/k_0$

f	Jacket Mode $\sigma d = 10^{-3}$	Jacket Mode $\sigma d = 10^{-2}$	Jacket Mode $\sigma d = 10^{-1}$
.2	759.7	240.2	75.98
.5	480.5	151.9	48.06
1.0	339.8	107.4	34.00
2.0	240.2	75.98	24.06
5.0	151.9	48.06	15.25
10.0	107.4	34.00	10.82
20.0	75.98	24.06	7.699
50.0	48.06	15.25	4.967
100.0	34.00	10.82	3.630

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Electromagnetic Transmission Through a Filled Slit in a Conducting Plane of Finite Thickness, TE Case

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***Abstract*—A solution is developed for computing the transmission characteristics of a slit in a conducting screen of finite thickness placed between two different media. The slit may be filled with lossy material while the two regions on either side of the screen are assumed lossless. A magnetic line source excitation is used (TE case) which is parallel to the axis of the slit. The equivalence principle is invoked to replace the two slit faces by equivalent magnetic current sheets on perfect electric conductors. Two coupled integral equations containing the magnetic currents as unknowns are then obtained and solved for by the method of moments. Pulses are used for the expansion and testing functions. Quantities computed are equivalent magnetic currents, the transmission coefficient, the gain pattern, and the normalized far field pattern.**

I. Introduction

THE PROBLEM of diffraction of plane waves through a slit in a perfect electric conductor of finite thickness has been studied by several investigators [1]–[5]. The most extensive investigation was that of Lehman [1], who used the analytic properties of finite Fourier transforms. The solution of Kashyap and Hamid [2] used a Wiener–Hopf and generalized matrix technique. Both of these solutions were done for the TM case (incident electric field parallel to slit axis). The solutions of Hongo [3] and of Neerhoff and Mur [4], were obtained by a numerical solution of coupled integral equations and were done for the TE case. A similar solution for the TM case was obtained by Wirgin [5]. In this paper, we use the method of moments to solve coupled integral equations similar in form to those derived in [4].

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This paper utilizes the generalized network formulation of coupling through apertures developed in [6] and [7] and extends these results to three regions coupled by two apertures. To accomplish this the equivalence principle is used to replace both faces of the slit by perfect conductors, each of which carry magnetic current sheets on both sides. The original problem is now broken up into three regions which are coupled by the postulated magnetic current sheets. The two half-space regions are loss free with arbitrary μ and ϵ and the medium in the slit is assumed lossy with arbitrary complex μ and ϵ.

Continuity of the tangential magnetic field is used to derive two coupled operator equations involving the equivalent magnetic currents as unknowns. These equations are put into matrix form using the method of moments, and solved by using standard matrix methods. The result can be interpreted in terms of a combination of "admittance matrices" computed separately for each region. This gives rise to a network interpretation of the problem which treats the unknown magnetic currents as port voltages and the excitation as port currents.

II. Problem Formulation

The original problem configuration is shown in Fig. 1. It consists of a perfect electric conductor of thickness d separating two regions a and c which may have different electrical properties. Coupling between the two regions occurs through a slit of width w filled with an arbitrarily lossy medium. The conductor is infinite in the z and y directions. The problem consists of three regions separated by two boundaries (the slit faces). Using the equivalence principle, the three regions can be separated by

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